13.3 Videos Guide

13.3a

- Arc length
  - $L = \int_a^b |\mathbf{r}'(t)|\, dt$

Exercises:

- Find the length of the curve $\mathbf{r}(t) = \langle 2t, t^2, \frac{1}{3}t \rangle, \qquad 0 \le t \le 1.$
- Find, correct to four decimal places, the length of the curve of intersection of the cylinder $4x^2 + y^2 = 4$ and the plane $x + y + z = 2$.

13.3b

- Arc length function
  - $s(t) = \int_a^t |\mathbf{r}'(u)|\, du$

Exercise:

- (a) Find the arc length function for the curve measured from the point P in the direction of increasing t and then reparameterize the curve with respect to arc length starting from P, and (b) find the point 4 units along the curve (in the direction of increasing t) from P.
  $\mathbf{r}(t) = e^t \sin t\, \mathbf{i} + e^t \cos t\, \mathbf{j} + \sqrt{t}e^t\mathbf{k}, \qquad P(0, 1, \sqrt{2})$

13.3c

- Curvature
  - For a vector function: $\kappa(t) = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$
  - For a plane curve in $\mathbb{R}^2$: $\kappa(x) = \frac{|f''(x)|}{[1 + [f'(x)]^2]^{3/2}}$
  - The radius of the osculating circle to a curve at a point is $\rho = \frac{1}{\kappa}$ at that point
- Development of the formula $\kappa(t) = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|}$

13.3d

- Development of the formula $\kappa(t) = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$

13.3e

- Unit vectors
  - The unit tangent vector: $\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$ (from Lesson 13.2)
  - The unit normal vector: $\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|}$
  - The binormal vector: $\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$

- Planes
    - The direction vector for the normal plane is the unit tangent vector $\mathbf{T}(t)$
    - The direction vector for the binormal plane is the binormal vector $\mathbf{B}(t)$

Exercises:

13.3f

- (a) Find the unit tangent and unit normal vectors $\mathbf{T}(t)$ and $\mathbf{N}(t)$.
  (b) Use $\kappa(t) = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|}$ to find the curvature.
  $\mathbf{t}(t) = \langle t^2, \sin t - t, \cos t + t \sin t \rangle,\ t > 0$

13.3g

- Use $\kappa(t) = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$ to find the curvature.
  $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + e^t\mathbf{k}$

13.3h

- Find equations of the normal plane and osculating plane of the curve at the given point.
  $x = \ln t, y = 2t, z = t^2, \qquad (0, 2, 1)$